弹塑性力学研究生核心课程

第六讲 黏弹性理论

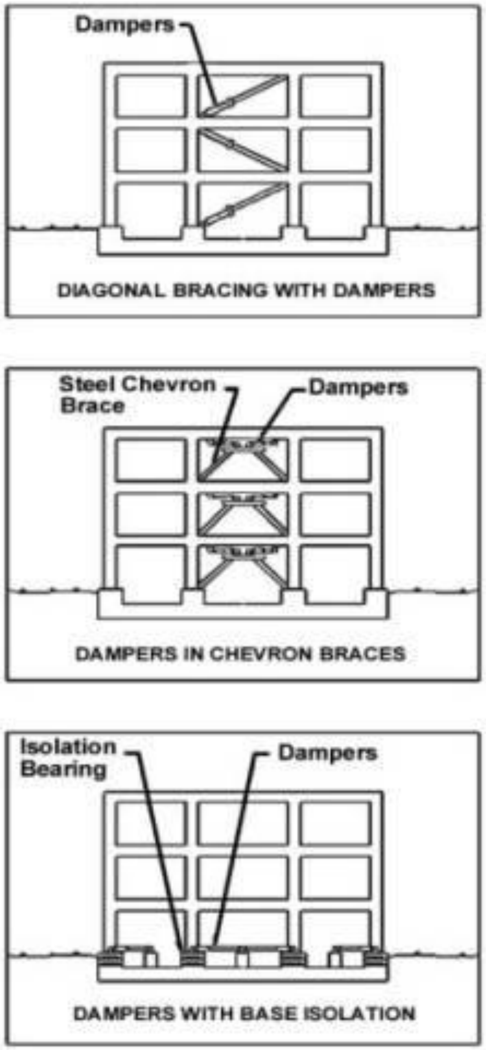
任晓丹

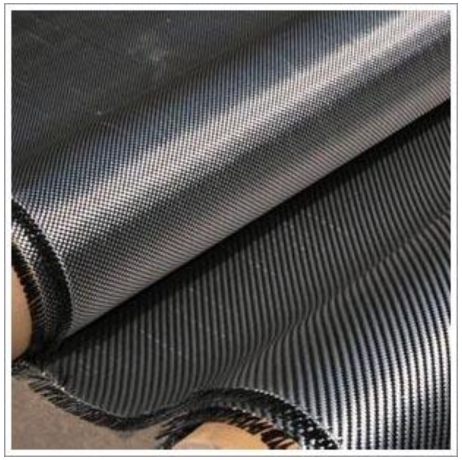
www.renxiaodan.com

rxdtj@tongji.edu.cn

同济大学土木工程学院

率相关元件、构件





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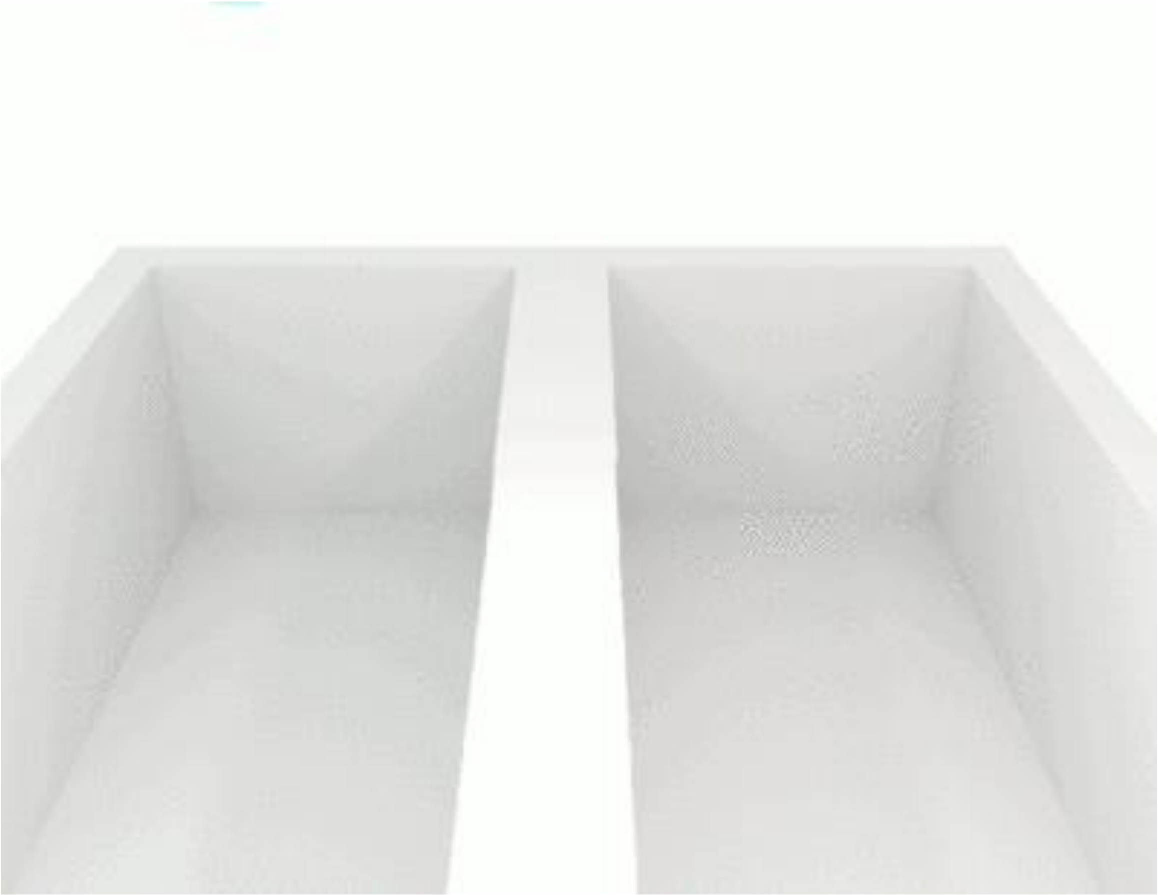
黏性（VISCOSITY）



黏性（VISCOSITY）

世界上时间最长的科学实验

The best known version of the experiment was started in 1927 by Professor Thomas Parnell of the University of Queensland in Brisbane, Australia, to demonstrate to students that some substances that appear solid are, in fact, very-high-viscosity fluids. Parnell poured a heated sample of pitch into a sealed funnel and allowed it to settle for three years. In 1930, the seal at the neck of the funnel was cut, allowing the pitch to start flowing. A glass dome covers the funnel and it is placed on display outside a lecture theatre. Large droplets form and fall over a period of about a decade. The eighth drop fell on 28 November 2000, allowing experimenters to calculate that the pitch has a viscosity approximately 230 billion (2.3×10^{11}) times that of water.

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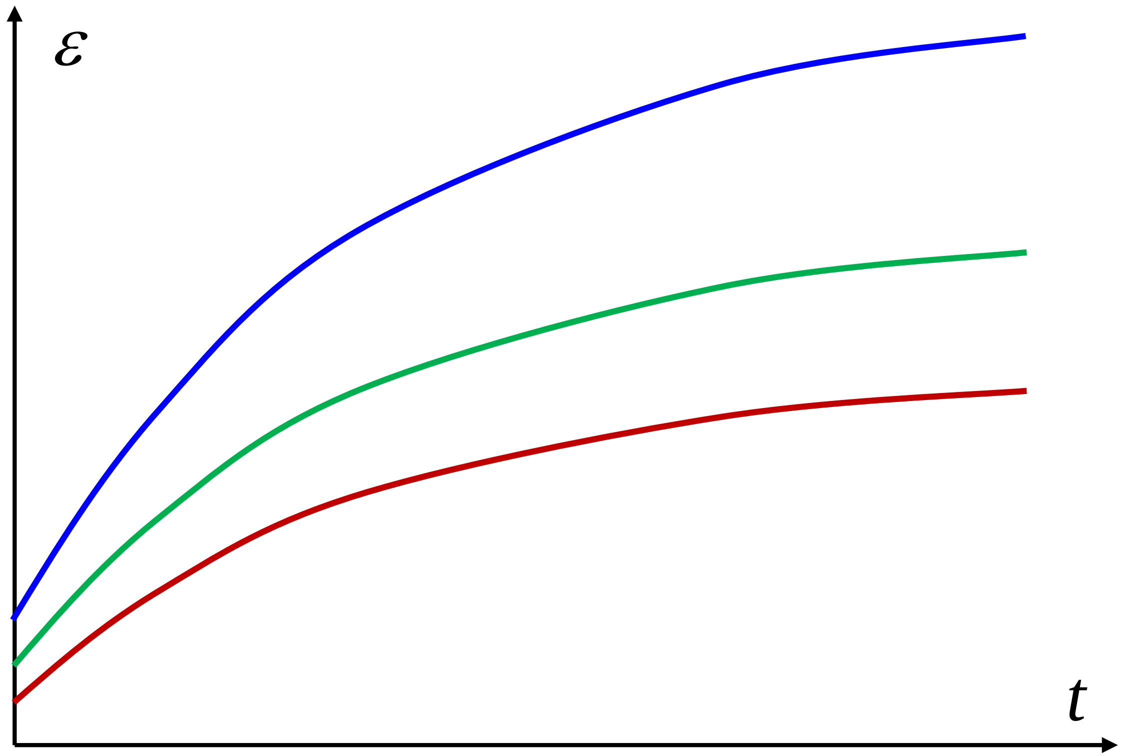
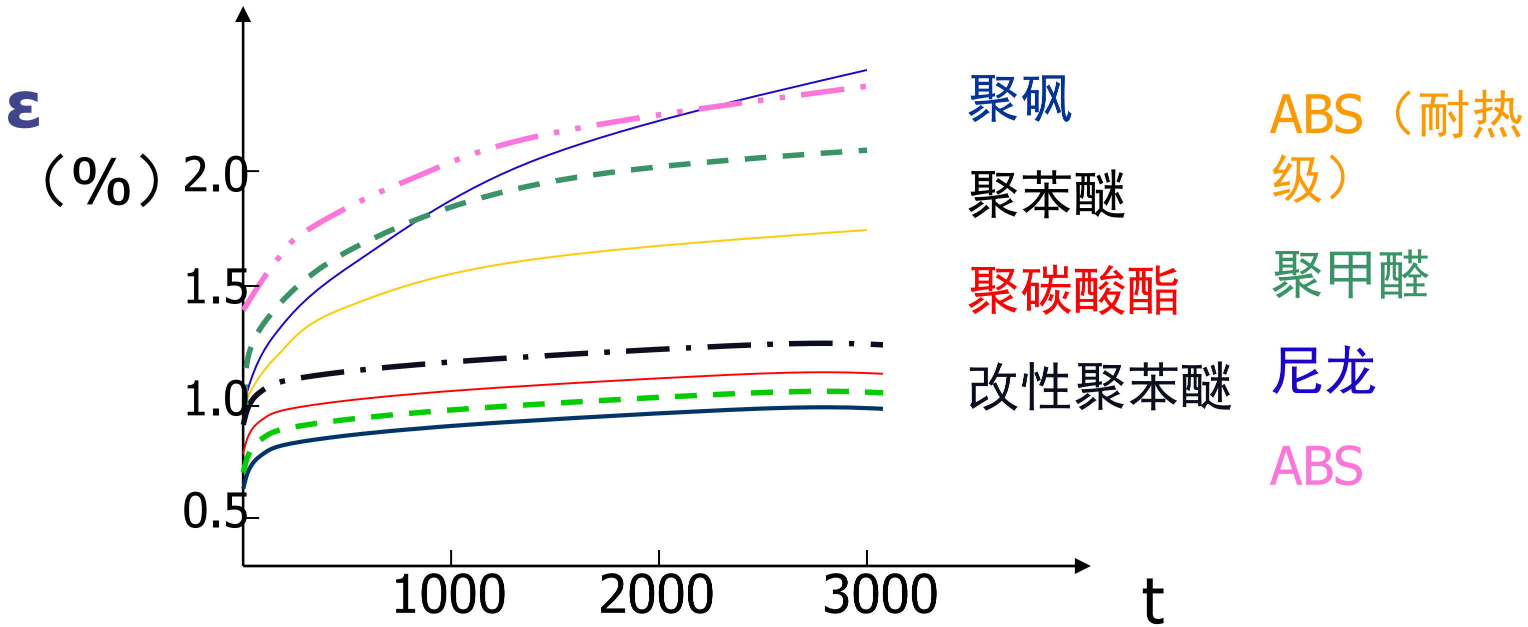
固体黏性



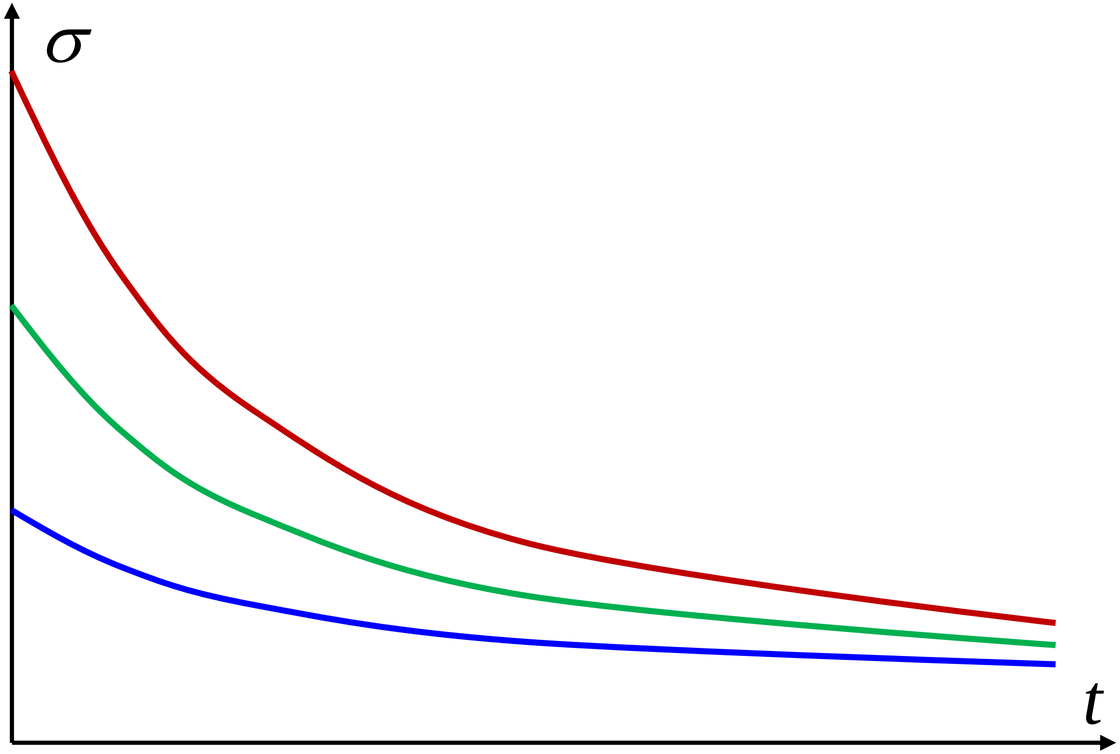
徐变（creep）：恒定应力作用下应变随时间持续增长

固体黏性



各类材料的徐变

固体黏性

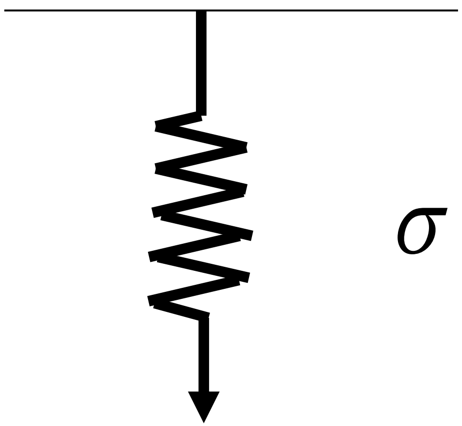


松弛（relaxation）：恒定应变（变形）作用下应力随时间持续降低

线性黏弹性模型

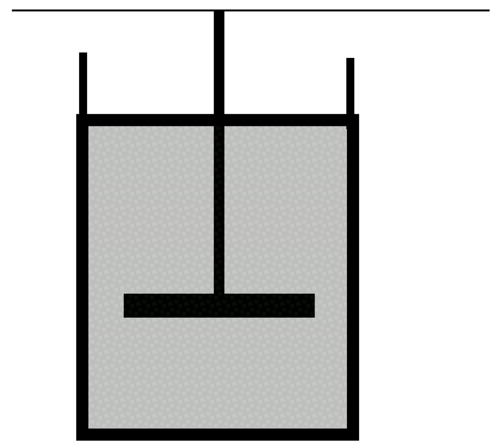
如一个符合虎克定律的弹簧能很好的描述理想弹性体：

一个具有一块平板浸没在一个充满粘度为η，符合牛顿流动定律的流体的小壶组成的粘壶，可以用来描述理想流体的力学行为.



$$\sigma = E\varepsilon$$

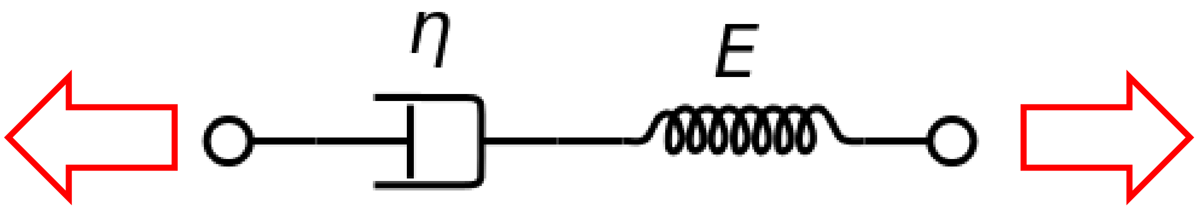
理想弹簧



$$\sigma = \eta \frac{d\varepsilon}{dt}$$

理想粘壶

MAXWELL模型



$$\begin{cases} \sigma = E\varepsilon_e \\ \sigma = \eta\dot{\varepsilon}_v \end{cases} \Rightarrow \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

对于松弛问题：

$$\dot{\varepsilon} = 0 \Rightarrow \frac{\dot{\sigma}}{E} = -\frac{\sigma}{\eta} \Rightarrow \begin{cases} \sigma = \sigma_0 e^{-\frac{E}{\eta}t} \\ E(t) = \sigma/\varepsilon_0 = E_0 e^{-\frac{E}{\eta}t} \end{cases}$$

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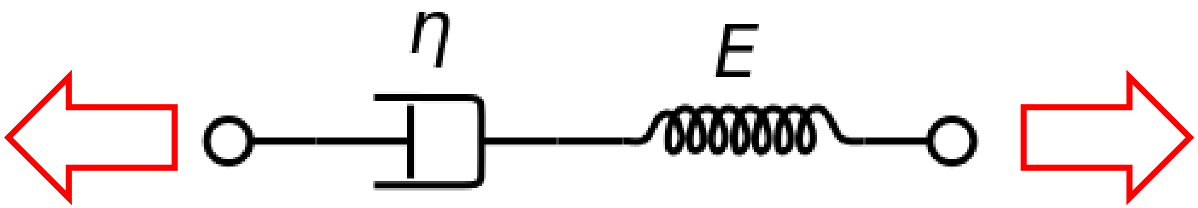
MAXWELL模型

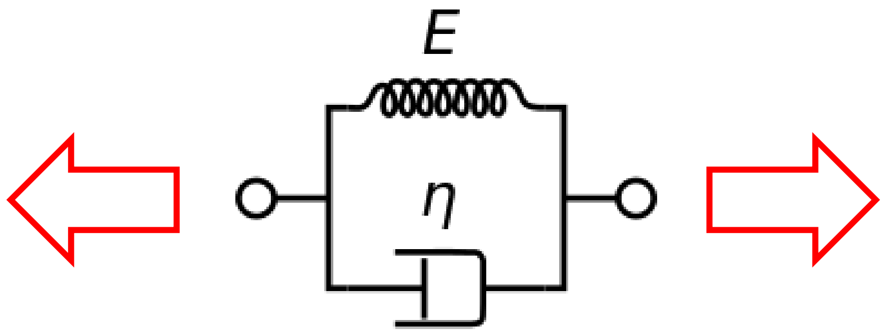


$$\begin{cases} \sigma = E\varepsilon_e \\ \sigma = \eta\dot{\varepsilon}_v \end{cases} \Rightarrow \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

通解：

$$\sigma = e^{-\frac{E}{\eta}t}\left[\sigma_0 + E\int_0^t \frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} e^{\frac{E}{\eta}\tau}\mathrm{d}\tau\right]$$

KELVIN模型

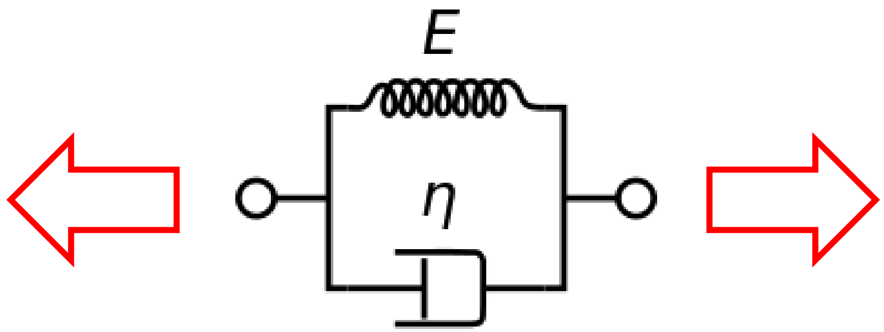


$$\frac{\sigma_e}{E} = \varepsilon, \quad \frac{\sigma_v}{\eta} = \dot{\varepsilon} \quad \Rightarrow \quad \sigma = E\varepsilon + \eta\dot{\varepsilon}$$

通解：

$$\varepsilon = e^{-\frac{E}{\eta}t}\left[\varepsilon_0 + \frac{1}{\eta}\int_0^t \sigma(\tau) e^{\frac{E}{\eta}\tau} \mathrm{d}\tau\right]$$

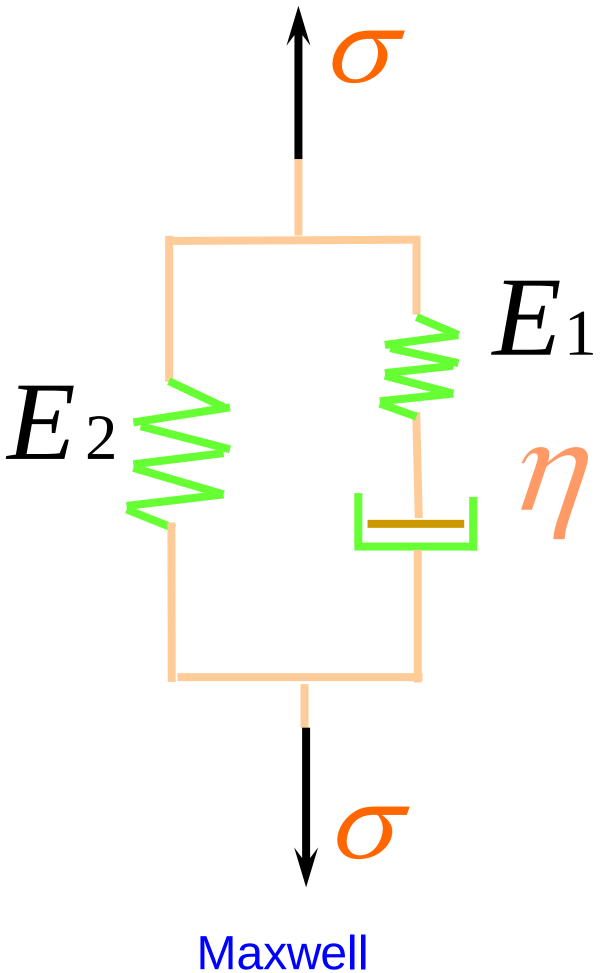
KELVIN模型



$$\varepsilon = e^{-\frac{E}{\eta}t}\left[\varepsilon_0 + \frac{1}{\eta}\int_0^t \sigma(\tau) e^{\frac{E}{\eta}\tau}\mathrm{d}\tau\right]$$

对于徐变问题： $\sigma = \sigma_0$ $\varepsilon = \varepsilon_0(1+e^{-\frac{E}{\eta}t})$

标准固体模型



$$\dot{\varepsilon} = \frac{\dot{\sigma}_1}{E_1} + \frac{\sigma_1}{\eta} \qquad \varepsilon = \frac{\sigma_2}{E_2}$$

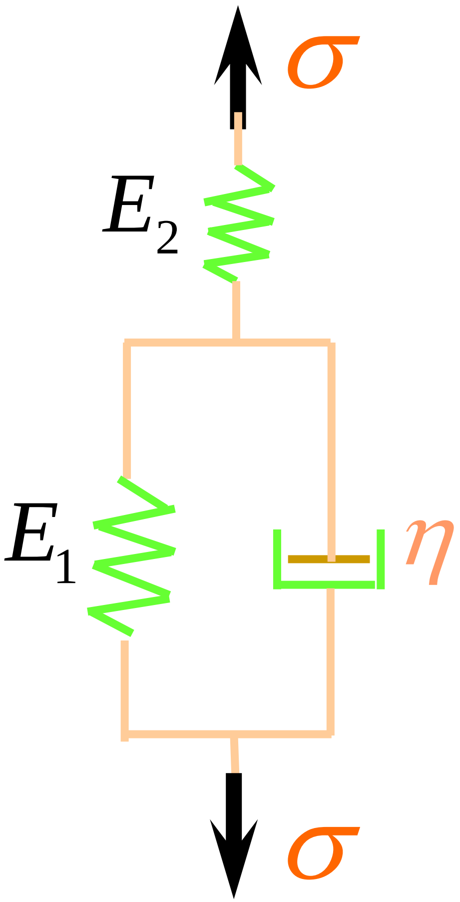
⇓

$$\dot{\varepsilon} = \frac{\dot{\sigma} - E_2\dot{\varepsilon}}{E_1} + \frac{\sigma - E_2\varepsilon}{\eta}$$

⇓

$$\left(1 + \frac{E_2}{E_1}\right)\dot{\varepsilon} + \frac{E_2}{\eta}\varepsilon = \frac{\dot{\sigma}}{E_1} + \frac{\sigma}{\eta}$$

标准固体模型

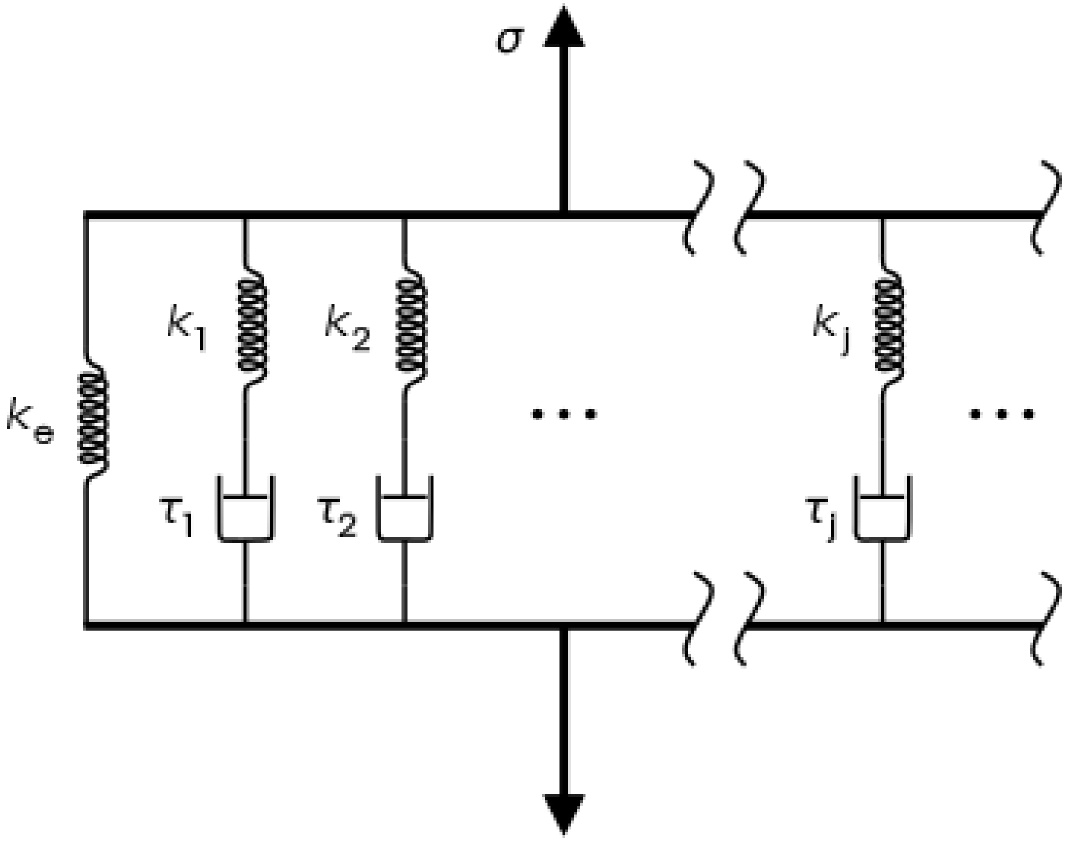


Kelvin

$$\sigma = E\varepsilon_1 + \eta\dot{\varepsilon}_1 \qquad \varepsilon_2 = \frac{\sigma}{E_2}$$

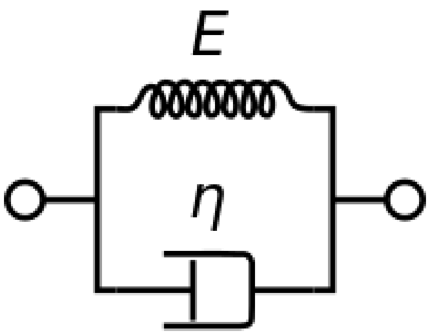
$$\frac{E_1 + E_2}{E_2}\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1\varepsilon + \eta\dot{\varepsilon}$$

广义MAXWELL模型



上述线性系统的解可用积分变换法建立，最通用的方法为拉普拉斯变换法，可参阅有关书籍，此处不再赘述。

黏（弹）性阻尼器



$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

令： $\varepsilon = A\cos(\omega t)$

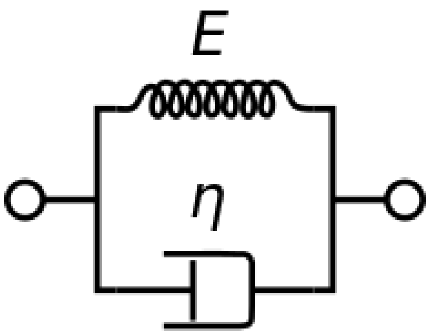
$$\sigma = E\varepsilon - \eta A\omega\sin(\omega t)$$

对粘滞阻尼器，可令： $E \approx 0$

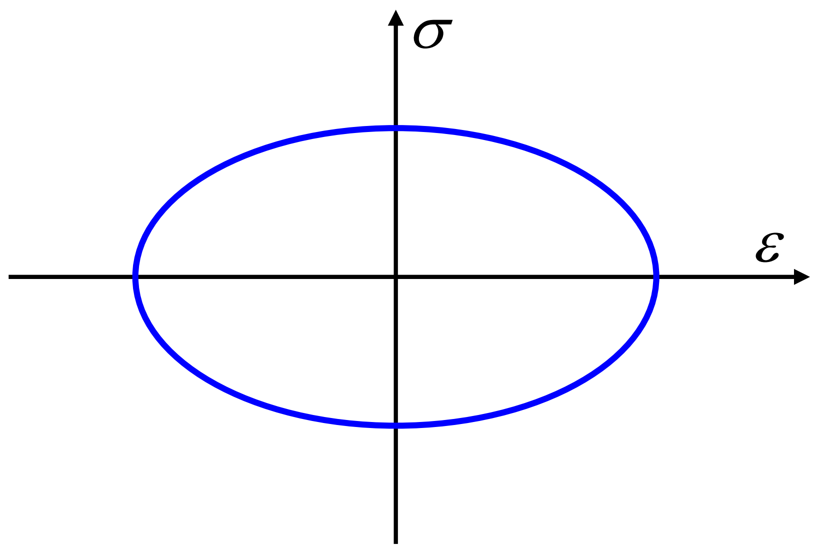
$$\varepsilon^2 + \left(\frac{\sigma}{\omega\eta}\right)^2 = A^2$$

黏（弹）性阻尼器

$$\varepsilon^2 + \left(\frac{\sigma}{\omega\eta}\right)^2 = A^2$$

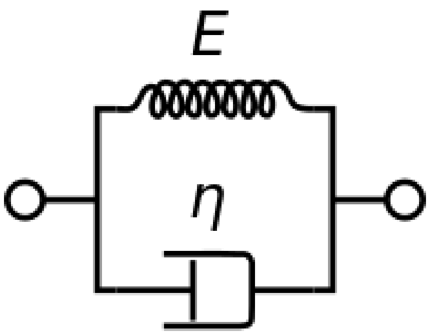


$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$



线性粘滞阻尼器的滞回曲线为椭圆

黏（弹）性阻尼器



$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

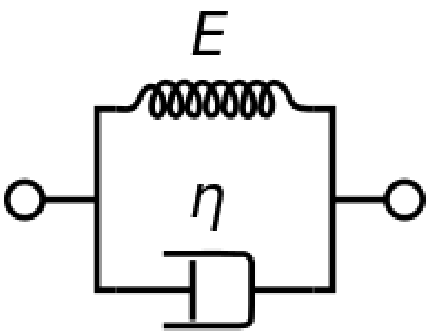
令： $\varepsilon = A\cos(\omega t)$

$$\sigma = E\varepsilon - \eta A\omega\sin(\omega t)$$

对粘弹阻尼器： $E > 0$

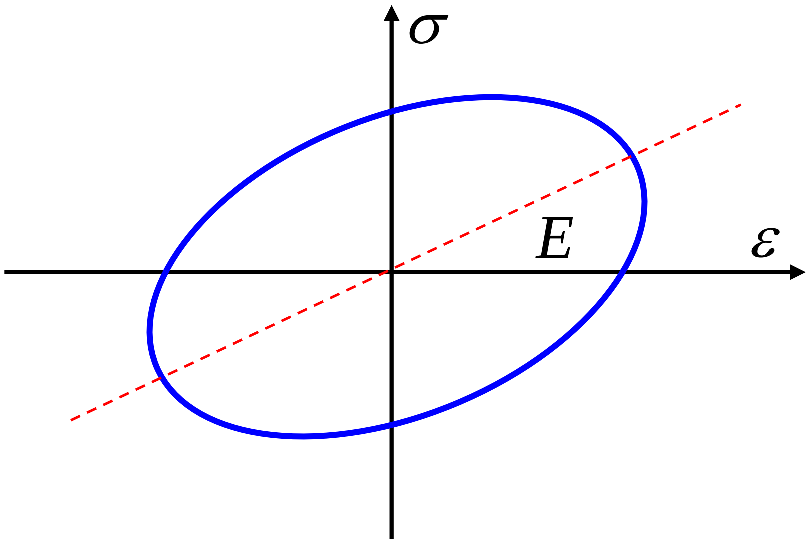
$$\varepsilon^2 + \left(\frac{\sigma - E\varepsilon}{\omega\eta}\right)^2 = A^2$$

黏（弹）性阻尼器



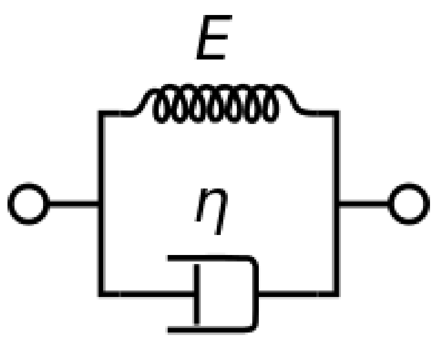
$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

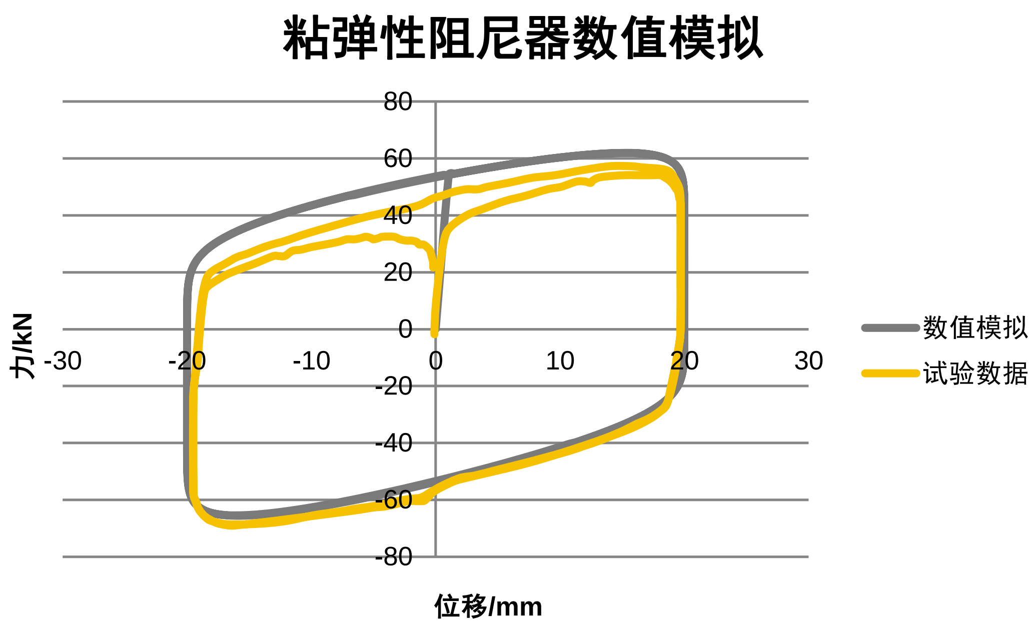
$$\varepsilon^2 + \left(\frac{\sigma - E\varepsilon}{\omega\eta}\right)^2 = A^2$$



线性粘弹性阻尼器的滞回曲线为斜椭圆

黏（弹）性阻尼器





$$\sigma = E\varepsilon + \eta\,\mathrm{sgn}(\dot{\varepsilon})\left|\dot{\varepsilon}\right|^{\alpha}$$

一般阻尼器，$0 < \alpha < 1$
较之椭圆更加饱满

数值算法怎么构造?

BOUC-WEN模型

$$m\ddot{u}(t)+c\dot{u}(t)+F(t)=f(t)$$

$$F(t)=ak_i u(t)+(1-a)k_i z(t)$$

$$\dot{z}(t)=A\dot{u}(t)-\beta\,|\dot{u}(t)|\,|z(t)|^{n-1}z(t)-\gamma\dot{u}(t)\,|z(t)|^{n}$$

简化

$$\dot{z}(t)=\dot{u}(t)\left\{A-\left[\beta\,\mathrm{sign}(z(t)\dot{u}(t))+\gamma\right]|z(t)|^{n}\right\}$$

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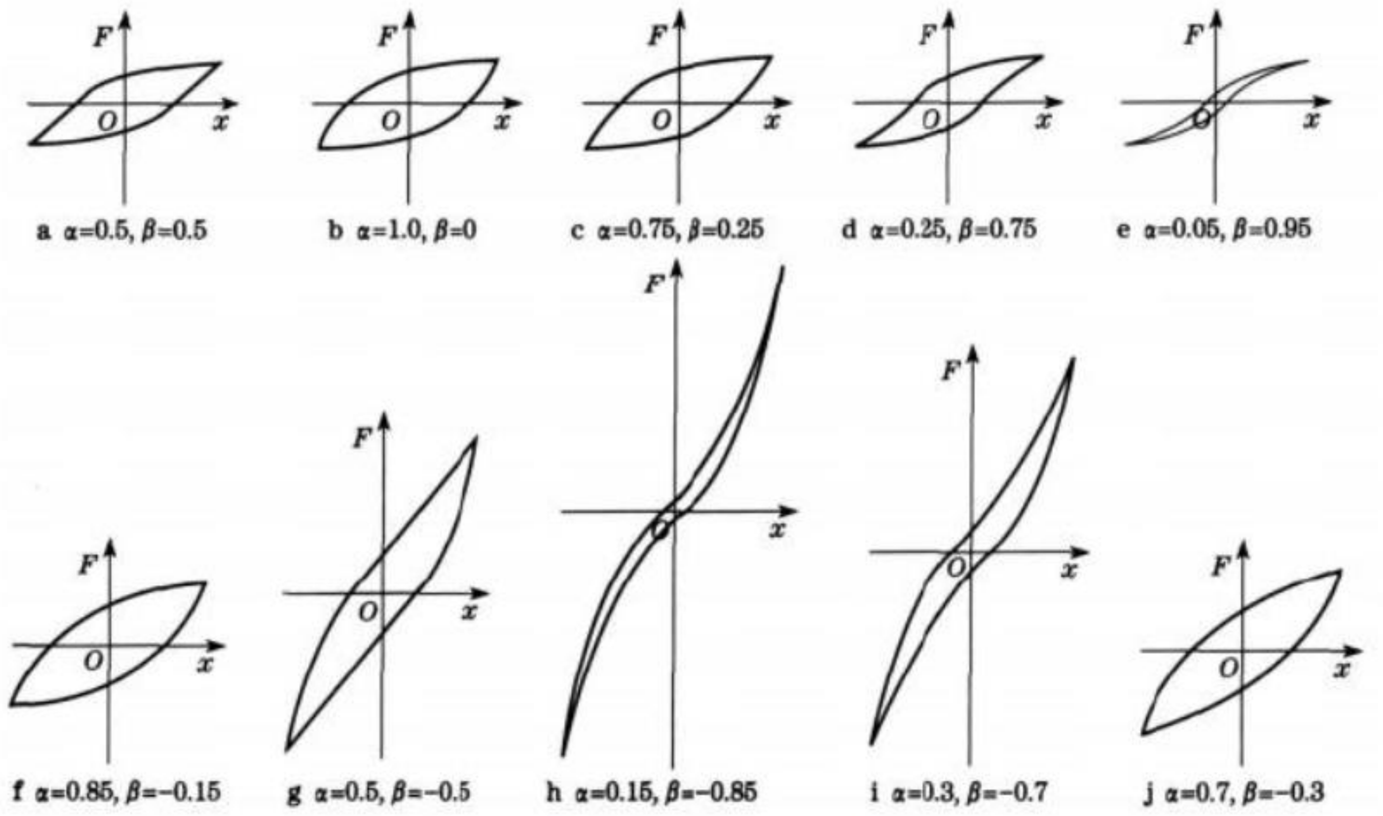
BOUC-WEN模型



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今天就到这里，

明天的事儿明天再说！

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